

بر اساس پروتکل‌های دوره‌های آموزشی آپتیم‌یار، به اشتراک‌گذاری محتوا و کدهای نرم‌افزاری منظر حقوقی ممنوع است و از منظر اخلاقی نارضایتی مدرس دوره و گروه آپتیم‌یار را به همراه دارد.

از توجه شما به پروتکل دوره‌های آموزشی آپتیم‌یار سپاسگزاریم.

دوره جامع آنلاین بهینه‌سازی استوار و برنامه‌ریزی در شرایط عدم قطعیت همراه با کدنویسی در نرم‌افزار (GAMS)

**Decision-Making under Uncertainty (Robust Optimization - Stochastic Programming - Fuzzy Programming)**

مدرس:

**دکتر علی پاپی (Ali Papi)**

تخصص شاخص: بهینه‌سازی و تحقیق در عملیات، علم تحلیل داده، تکنیک‌های تجزیه و روش‌های حل دقیق، بهینه‌سازی استوار داده‌محور، هوش محاسباتی و الگوریتم‌های فراابتکاری، نظریه بازی، بهینه‌سازی چندهدفه و تصمیم‌گیری چندمعیاره

Optimization & Operations Research, Data Analytics, Computational Intelligence & Metaheuristics, Decomposition Techniques & Exact Methods, Data-Driven Robust Optimization, Game Theory, Multi Criteria Decision Making

RobustMODM

Robust Payoff

Robust TH

Robust TH SensitivityAnalysis



**اخطار:** بر اساس پروتکل‌های دوره‌های آموزشی آپتیم‌یار، به اشتراک‌گذاری محتوا و کدهای نرم‌افزاری منظر حقوقی ممنوع است و از منظر اخلاقی نارضایتی مدرس دوره و گروه آپتیم‌یار را به همراه دارد.

باز توجه شما به پروتکل دوره‌های آموزشی آپتیم‌یار بسیار سپاسگزاریم.

## **RobustMODM**

Sets

$/j \ x \ /j1*j100$

$/k \ y \ /k1*k20$

$/i \ \text{cons} \ /i1*i50$

Parameters

'c(j)'nominal

f(k)

r(j)

'a(i,j)'nominal

d(i,k)

b(i)

h(k)

'e'nominal

;

; (10,20)c(j) = uniform

; (0,50)r(j) = uniform

; (700,1000)f(k) = uniform

; (2,8)a(i,j) = uniform

; (300,500)d(i,k) = uniform

; (1500,2500)b(i) = uniform

; (1,3)h(k) = uniform

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```
;e = 10
```

```
;
```

Parameters

```
PR_C(j)
```

```
PR_a(i,j)
```

```
PR_e
```

```
;
```

```
;PR_C(j) = 0.30
```

```
;PR_a(i,j) = 0.50
```

```
;PR_e = 0.60
```

Scalars

```
Gamma_o
```

```
Gamma_c1
```

```
Gamma_c2
```

```
;
```

```
;Gamma_o = sqrt(card(j))
```

```
;Gamma_c1 = sqrt(card(i)*card(j))
```

```
;Gamma_c2 = 1
```

Positive Variables

```
x(j)
```

```
;
```



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Binary Variables

y(k)

;

Free Variables

"Z1 "min

"Z2 "max

;

Equations

obj1

obj2

obj\_RC

cons1

cons1\_RC

cons2

cons\_add

;

Positive variables

p\_o(j)

q\_o

p\_c1(i,j)

;q\_c1(i)

;



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min\*

; obj1.. Z1 =g= sum(j,c(j)\*x(j)) + sum(k,f(k)\*y(k)) + sum(j,p\_o(j)) + Gamma\_o\*q\_o

;obj\_RC(j).. p\_o(j) + q\_o =g= x(j)\*c(j)\*PR\_c(j)

max\*

;obj2.. Z2 =e= sum(j,r(j)\*x(j))

; cons1(i).. sum(j,a(i,j)\*x(j)) + sum(k,d(i,k)\*y(k)) - (sum(j,p\_c1(i,j)) + Gamma\_c1\*q\_c1(i)) =g= b(i)

; cons\_add(i).. sum(j,a(i,j)\*(1-PR\_a(i,j))\*x(j)) =l= b(i)

;cons1\_RC(i,j).. p\_c1(i,j) + q\_c1(i) =g= x(j)\*a(i,j)\* PR\_a(i,j)

; cons2.. sum(k,h(k)\*y(k)) =l= e - Gamma\_c2\*e\*PR\_e

Model BS\_RC

/

obj1

obj2

obj\_RC

cons1

cons1\_RC

cons2

cons\_add

/

OptimYar

;

Options

MIP = CPLEX

OPTCR =0

RESLIM = 100

;

; Solve BS\_RC us MIP min Z1

Display

"Focus on min Z1"

z1.1

z2.1

x.1

y.1

;

; Solve BS\_RC us MIP max Z2

Display

"Focus on max Z2"

z1.1

z2.1

x.1

y.1

;



OptimYar

**Robust Payoff**

\*\*\*\*\*

\*\*\*\*\* The Best Payoff Matrix

\*\*\*\*\*

Set Objs

/

of1

of2

/

;

Alias(Objs,of)

;

Parameter

Payoff(of,of)

Max\_o(of)

Min\_o(of)

R\_o(of)

;

\*\*\*\*\* Problem Formulation/Modeling \*\*\*\*\*

Sets

j x /j1\*j100/

k y /k1\*k20/

i cons /i1\*i50/



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Parameters

$c(j)$  'nominal'

$f(k)$

$r(j)$

$a(i,j)$  'nominal'

$d(i,k)$

$b(i)$

$h(k)$

$e$  'nominal'

;

$c(j) = \text{uniform}(10,20);$

$r(j) = \text{uniform}(0,50);$

$f(k) = \text{uniform}(700,1000);$

$a(i,j) = \text{uniform}(2,8);$

$d(i,k) = \text{uniform}(300,500);$

$b(i) = \text{uniform}(1500,2500);$

$h(k) = \text{uniform}(1,3);$

$e = 10;$

;

Parameters

$PR\_C(j)$

$PR\_a(i,j)$

$PR\_e$

;

$PR\_C(j) = 0.30;$

$PR\_a(i,j) = 0.50;$



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PR\_e = 0.60;

Scalars

Gamma\_o

Gamma\_c1

Gamma\_c2

;

Gamma\_o = sqrt(card(j));

Gamma\_c1 = sqrt(card(i)\*card(j));

Gamma\_c2 = 1;

Positive Variables

x(j)

;

Binary Variables

y(k)

;

Free Variables

Z1 "min"

Z2 "max"

;



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Equations

obj1

obj2

obj\_RC

cons1

cons1\_RC

cons2

cons\_add

;

Positive variables

p\_o(j)

q\_o

p\_c1(i,j)

q\_c1(i);

;

\*min

obj1.. Z1 =g= sum(j,c(j)\*x(j)) + sum(k,f(k)\*y(k)) + sum(j,p\_o(j)) + Gamma\_o\*q\_o ;

obj\_RC(j).. p\_o(j) + q\_o =g= x(j)\*c(j)\*PR\_c(j);

\*max

obj2.. Z2 =e= sum(j,r(j)\*x(j));

cons1(i).. sum(j,a(i,j)\*x(j)) + sum(k,d(i,k)\*y(k)) - (sum(j,p\_c1(i,j)) + Gamma\_c1\*q\_c1(i)) =g= b(i) ;

cons\_add(i).. sum(j,a(i,j)\*(1-PR\_a(i,j))\*x(j)) =l= b(i) ;

cons1\_RC(i,j).. p\_c1(i,j) + q\_c1(i) =g= x(j)\*a(i,j)\* PR\_a(i,j);

cons2.. sum(k,h(k)\*y(k)) =l= e - Gamma\_c2\*e\*PR\_e ;

Model BS\_RC

/

obj1

obj2

obj\_RC

cons1

cons1\_RC

cons2

cons\_add

/

;

Options

MIP = CPLEX

OPTCR =0

RESLIM = 100

;



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\*\*\*\*\* Find the "Payoff" matrix\*\*\*\*\*

Solve BS\_RC us MIP min Z1 ;

Payoff('of1','of1') = Z1.1;

Solve BS\_RC us MIP max Z2 ;

Payoff('of2','of2') = Z2.1;

Z2.fx=Payoff('of2','of2');

Solve BS\_RC us MIP min Z1 ;

Payoff('of1','of2') = Z1.1;

Z2.lo=-inf ;

Z2.up=inf ;

Z1.fx=Payoff('of1','of1');

Solve BS\_RC us MIP max Z2 ;

Payoff('of2','of1') = Z2.1;

Z1.lo=-inf ;

Z1.up=inf ;

\*\*\*\*\* Min Max Range

Min\_o(of)= smin[objs,payoff(of,objs)];

Max\_o(of)= smax[objs,payoff(of,objs)];

R\_o(of)= Max\_o(of) - Min\_o(of) ;

\*\*\*\*\*

Display

Payoff

Min\_o

Max\_o

R\_o



## **Robust TH**

\* MODM Method (TH) A. Papi

\* Trading-off between Compensatory (Norm 1) and Non-compensatory Solutions (Norm inf)

\*\*\*\*\* Problem Formulation/Modeling \*\*\*\*\*

Sets

$j \in J = \{1, \dots, 100\}$   
 $k \in K = \{1, \dots, 20\}$   
 $i \in I = \{1, \dots, 50\}$

Parameters

$c(j)$  'nominal'  
 $f(k)$   
 $r(j)$   
 $a(i,j)$  'nominal'  
 $d(i,k)$   
 $b(i)$   
 $h(k)$   
 $e$  'nominal'  
;

$c(j) = \text{uniform}(10,20);$   
 $r(j) = \text{uniform}(0,50);$   
 $f(k) = \text{uniform}(700,1000);$

```
a(i,j) = uniform(2,8);  
d(i,k) = uniform(300,500);  
b(i)   = uniform(1500,2500);  
h(k)   = uniform(1,3);  
e      = 10;  
;
```

Parameters

```
PR_C(j)
```

```
PR_a(i,j)
```

```
PR_e
```

```
;
```

```
PR_C(j) = 0.30;
```

```
PR_a(i,j) = 0.50;
```

```
PR_e = 0.60;
```

Scalars

```
Gamma_o
```

```
Gamma_c1
```

```
Gamma_c2
```

```
;
```

```
Gamma_o = sqrt(card(j));
```

```
Gamma_c1 = sqrt(card(i)*card(j));
```

```
Gamma_c2 = 1;
```



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Positive Variables

x(j)

;

Binary Variables

y(k)

;

Free Variables

Z1 "min"

Z2 "max"

;

Equations

obj1

obj2

obj\_RC

cons1

cons1\_RC

cons2

cons\_add

;

Positive variables

p\_o(j)

q\_o



p\_c1(i,j)

q\_c1(i);

;

\*min

obj1.. Z1 =g= sum(j,c(j)\*x(j)) + sum(k,f(k)\*y(k)) + sum(j,p\_o(j)) + Gamma\_o\*q\_o ;

obj\_RC(j).. p\_o(j) + q\_o =g= x(j)\*c(j)\*PR\_c(j);

\*max

obj2.. Z2 =e= sum(j,r(j)\*x(j));

cons1(i).. sum(j,a(i,j)\*x(j)) + sum(k,d(i,k)\*y(k)) - (sum(j,p\_c1(i,j)) + Gamma\_c1\*q\_c1(i)) =g= b(i) ;

cons\_add(i).. sum(j,a(i,j)\*(1-PR\_a(i,j))\*x(j)) =l= b(i) ;

cons1\_RC(i,j).. p\_c1(i,j) + q\_c1(i) =g= x(j)\*a(i,j)\* PR\_a(i,j);

cons2.. sum(k,h(k)\*y(k)) =l= e - Gamma\_c2\*e\*PR\_e ;

Model BS\_RC

/

obj1

```
obj2
obj_RC
cons1
cons1_RC
cons2
cons_add
/
;

Options
MIP = CPLEX
OPTCR =0
RESLIM = 100
;

*****
***** The Best Payoff Matrix
*****

Set Objs
/
of1
of2
/
;
Alias(Objs,of)
;

Parameter
Payoff(of,of)
```



Max\_o(of)

Min\_o(of)

R\_o(of)

;

\*\*\*\*\* Find the "Payoff" matrix\*\*\*\*\*

Solve BS\_RC us MIP min Z1 ;

Payoff('of1','of1') = Z1.1;

Solve BS\_RC us MIP max Z2 ;

Payoff('of2','of2') = Z2.1;

Z2.fx=Payoff('of2','of2');

Solve BS\_RC us MIP min Z1 ;

Payoff('of1','of2') = Z1.1;

Z2.lo=-inf ;

Z2.up=inf ;

Z1.fx=Payoff('of1','of1');

Solve BS\_RC us MIP max Z2 ;

Payoff('of2','of1') = Z2.1;

Z1.lo=-inf ;

Z1.up=inf ;

\*\*\*\*\* Min Max Range\*\*\*\*\*

Min\_o(of)= smin[objs,payoff(of,objs)];

Max\_o(of)= smax[objs,payoff(of,objs)];

R\_o(of)= Max\_o(of) - Min\_o(of) ;

\*\*\*\*\*

Display

Payoff

Min\_o

Max\_o

R\_o

\*\*\*\*\*

\*\* TH Method

Positive Variable

N1

N2

;

Equations

Norm\_of1 'min : the least is the best'

Norm\_of2 'max : the least is the best'

;

Norm\_of1.. N1 =e= [Z1 - min\_o('of1')]/R\_o('of1');

Norm\_of2.. N2 =e= [max\_o('of2') - Z2]/R\_o('of2');

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Positive Variable

Lp1

Lpinf

;

Equations

E\_Lp1

E\_Lpinf\_of1

E\_Lpinf\_of2

;

E\_Lp1 .. Lp1 =e= (N1+N2)/2;

E\_Lpinf\_of1.. Lpinf =g= N1;

E\_Lpinf\_of2.. Lpinf =g= N2;

\*\*\*\*\* TH Measure

Equations

Me\_TH

;

Free Variable Z\_TH 'min'

;

Scalars

w1 /0.999/

w2 /0.001/

;

1

Me\_TH .. Z\_TH =e= w1\*Lp1 + w2\*Lpinf;

The logo for OptimYar features a large, stylized blue gear on the left and a blue wave-like shape on the right. The word "OptimYar" is written in a large, blue, sans-serif font at the bottom center, with the "Y" being significantly larger than the other letters.

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Model Robust\_TH

/

BS\_RC

Norm\_of1

Norm\_of2

E\_Lp1

E\_Lpinf\_of1

E\_Lpinf\_of2

Me\_TH

/

;

Solve Robust\_TH us MIP min Z\_TH;

\*\*\*\*\* Final Result/Output\*\*\*\*\*

Display

z1.1

z2.1

N1.1

N2.1

Lp1.1

Lpinf.1

x.1

y.1

;



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## **Robust TH Sensitivity Analysis**

\* MODM Method (TH) A. Papi

\* Trading-off between Compensatory (Norm 1) and Non-compensatory Solutions (Norm inf)

\*\*\*\*\* Problem Formulation/Modeling \*\*\*\*\*

Sets

$j \in J = \{1, \dots, 100\}$

$k \in K = \{1, \dots, 20\}$

$i \in I = \{1, \dots, 50\}$

Parameters

$c(j)$  'nominal'

$f(k)$

$r(j)$

$a(i,j)$  'nominal'

$d(i,k)$

$b(i)$

$h(k)$

$e$  'nominal'

;

$c(j) = \text{uniform}(10,20);$

$r(j) = \text{uniform}(0,50);$

$f(k) = \text{uniform}(700,1000);$



```
a(i,j) = uniform(2,8);  
d(i,k) = uniform(300,500);  
b(i)   = uniform(1500,2500);  
h(k)   = uniform(1,3);  
e      = 10;  
;
```

Parameters

PR\_C(j)

PR\_a(i,j)

PR\_e

;

PR\_C(j) = 0.30;

PR\_a(i,j) = 0.50;

PR\_e = 0.60;

Scalars

Gamma\_o

Gamma\_c1

Gamma\_c2

;

Gamma\_o = sqrt(card(j));

Gamma\_c1 = sqrt(card(i)\*card(j));

Gamma\_c2 = 1;



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Positive Variables

x(j)

;

Binary Variables

y(k)

;

Free Variables

Z1 "min"

Z2 "max"

;

Equations

obj1

obj2

obj\_RC

cons1

cons1\_RC

cons2

cons\_add

;

Positive variables

p\_o(j)

q\_o



p\_c1(i,j)

q\_c1(i);

;

\*min

obj1.. Z1 =g= sum(j,c(j)\*x(j)) + sum(k,f(k)\*y(k)) + sum(j,p\_o(j)) + Gamma\_o\*q\_o ;

obj\_RC(j).. p\_o(j) + q\_o =g= x(j)\*c(j)\*PR\_c(j);

\*max

obj2.. Z2 =e= sum(j,r(j)\*x(j));

cons1(i).. sum(j,a(i,j)\*x(j)) + sum(k,d(i,k)\*y(k)) - (sum(j,p\_c1(i,j)) + Gamma\_c1\*q\_c1(i)) =g= b(i) ;

cons\_add(i).. sum(j,a(i,j)\*(1-PR\_a(i,j))\*x(j)) =l= b(i) ;

cons1\_RC(i,j).. p\_c1(i,j) + q\_c1(i) =g= x(j)\*a(i,j)\* PR\_a(i,j);

cons2.. sum(k,h(k)\*y(k)) =l= e - Gamma\_c2\*e\*PR\_e ;

Model BS\_RC

/

obj1

```
obj2
obj_RC
cons1
cons1_RC
cons2
cons_add
/
;

Options
MIP = CPLEX
OPTCR =0
RESLIM = 100
;

*****
***** The Best Payoff Matrix
*****

Set Objs
/
of1
of2
/
;
Alias(Objs,of)
;

Set Iter /iter1*iter5/
```



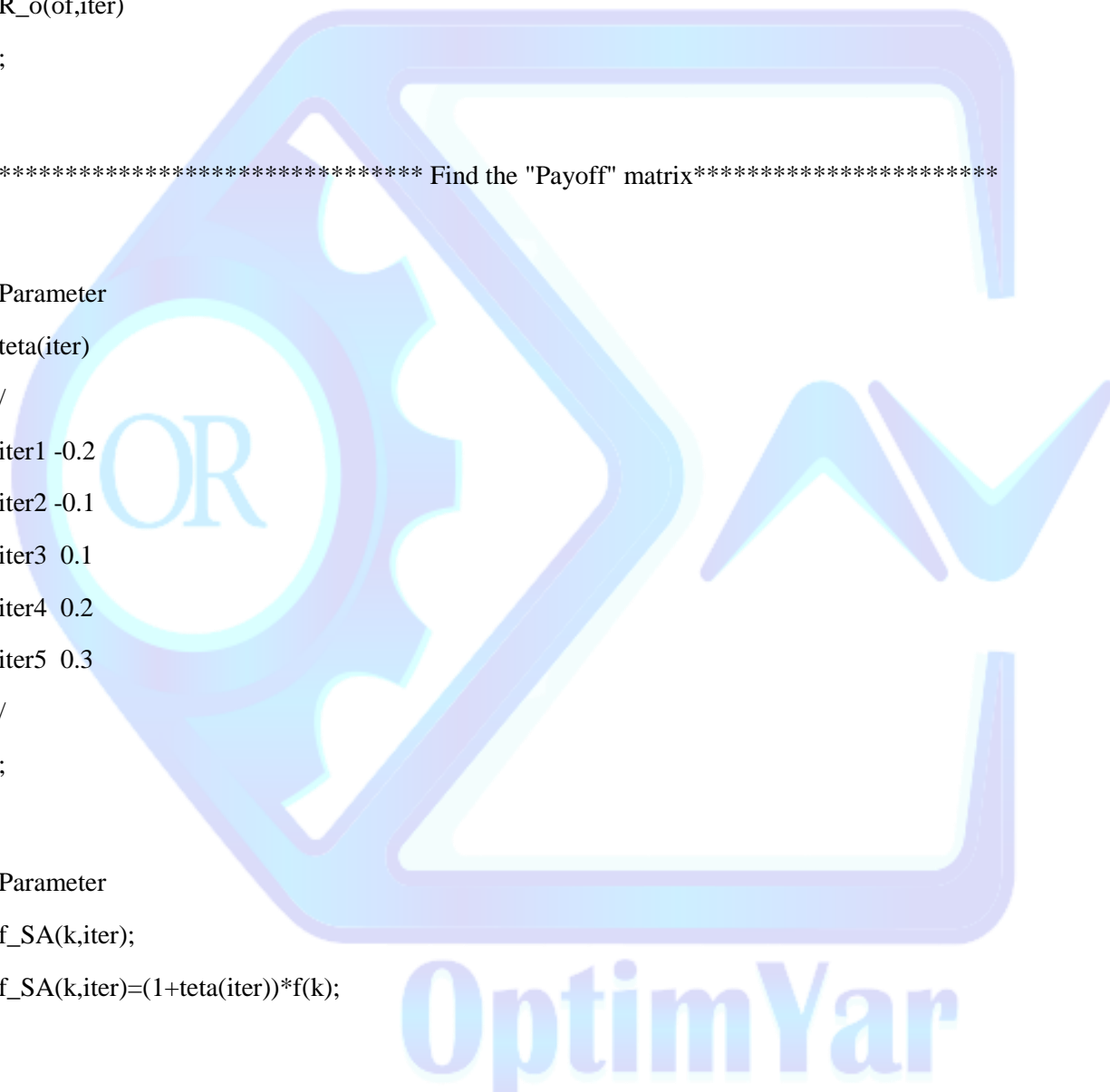
```
Parameter  
Payoff(of,of,iter)  
Max_o(of,iter)  
Min_o(of,iter)  
R_o(of,iter)  
;
```

```
***** Find the "Payoff" matrix*****
```

```
Parameter  
teta(iter)  
/  
iter1 -0.2  
iter2 -0.1  
iter3 0.1  
iter4 0.2  
iter5 0.3  
/  
;
```

```
Parameter  
f_SA(k,iter);  
f_SA(k,iter)=(1+teta(iter))*f(k);
```

```
Loop(iter,  
  
f(k)=f_SA(k,iter);
```



Solve BS\_RC us MIP min Z1 ;

Payoff('of1','of1',iter) = Z1.l;

Solve BS\_RC us MIP max Z2 ;

Payoff('of2','of2',iter) = Z2.l;

Z2.fx=Payoff('of2','of2',iter);

Solve BS\_RC us MIP min Z1 ;

Payoff('of1','of2',iter) = Z1.l;

Z2.lo=-inf ;

Z2.up=inf ;

Z1.fx=Payoff('of1','of1',iter);

Solve BS\_RC us MIP max Z2 ;

Payoff('of2','of1',iter) = Z2.l;

Z1.lo=-inf ;

Z1.up=inf ;

)

;

\*\*\*\*\* Min Max Range

Min\_o(of,iter)= smin[objs,payoff(of,objs,iter)];

Max\_o(of,iter)= smax[objs,payoff(of,objs,iter)];

R\_o(of,iter)= Max\_o(of,iter) - Min\_o(of,iter) ;

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\*\*\*\*\*

Display

Payoff

Min\_o

Max\_o

R\_o

\*\*\*\*\*

\*\* TH Method

Parameters

min\_o\_iter(of)

max\_o\_iter(of)

R\_o\_iter(of)

;

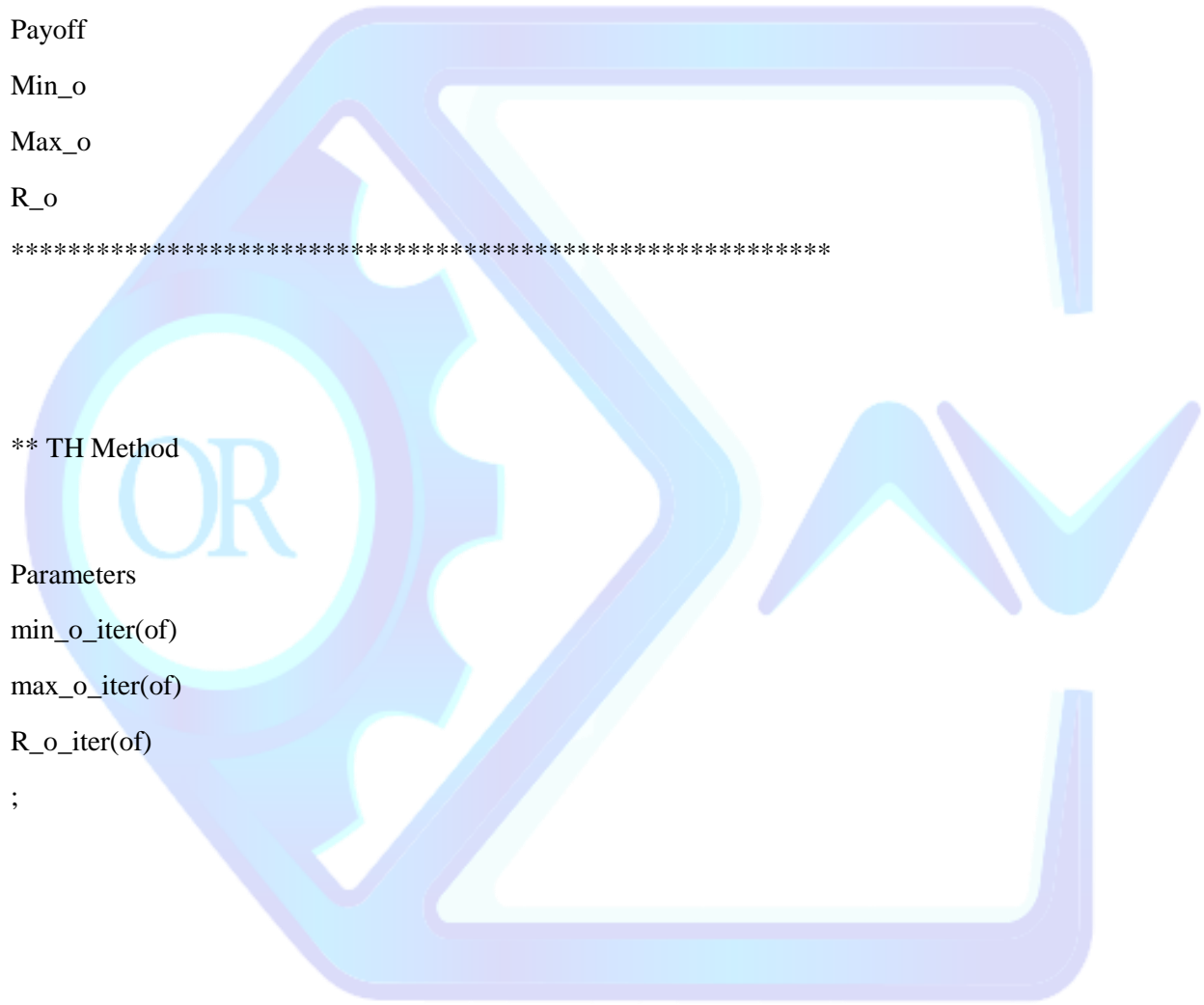
Positive Variable

N1

N2

;

Equations



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Norm\_of1 'min : the least is the best'

Norm\_of2 'max : the least is the best'

;

Norm\_of1.. N1 =e= [Z1 - min\_o\_iter('of1')]/R\_o\_iter('of1');

Norm\_of2.. N2 =e= [max\_o\_iter('of2') - Z2]/R\_o\_iter('of2');

Positive Variable

Lp1

Lpinf

;

Equations

E\_Lp1

E\_Lpinf\_of1

E\_Lpinf\_of2

;

E\_Lp1 .. Lp1 =e= [N1+N2]/2;

E\_Lpinf\_of1.. Lpinf =g= N1;

E\_Lpinf\_of2.. Lpinf =g= N2;

\*\*\*\*\* TH Measure

Equations

Me\_TH

;

Free Variable Z\_TH 'min'

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;

Scalars

w1 /0.999/

w2 /0.001/

;

Me\_TH .. Z\_TH =e= w1\*Lp1 + w2\*Lpinf;

Model Robust\_TH

/

BS\_RC

Norm\_of1

Norm\_of2

E\_Lp1

E\_Lpinf\_of1

E\_Lpinf\_of2

Me\_TH

/

;

Loop(iter,

min\_o\_iter(of)=min\_o(of,iter);

max\_o\_iter(of)=max\_o(of,iter);

R\_o\_iter(of)=R\_o(of,iter);

Solve Robust\_TH us MIP min Z\_TH;



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\*\*\*\*\* Final Result/Output\*\*\*\*\*

Display

'iter'

iter

z1.1

z2.1

N1.1

N2.1

Lp1.1

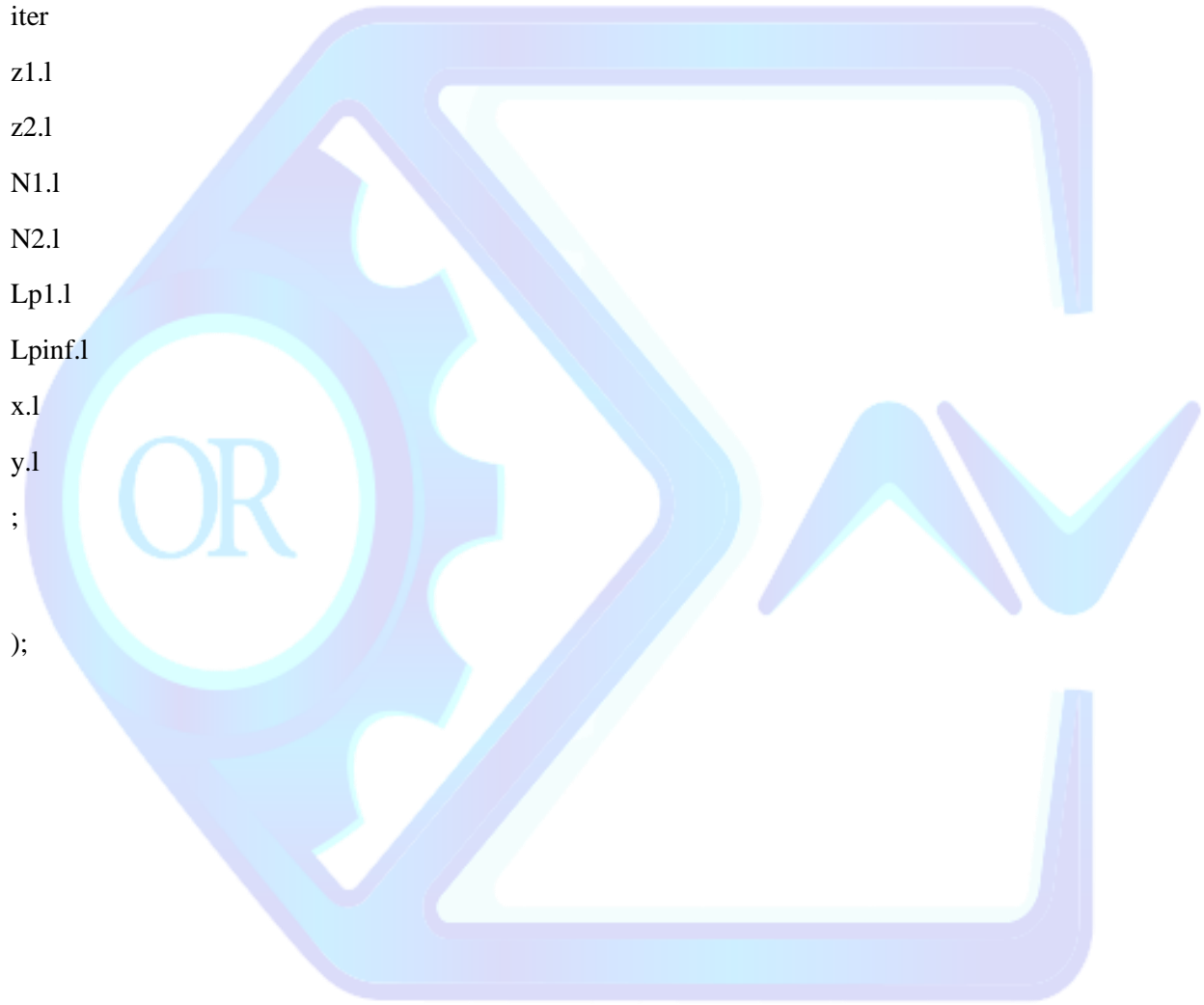
Lpinf.1

x.1

y.1

;

);



**OptimYar**

دوره جامع آنلاین بهینه‌سازی استوار و برنامه‌ریزی در شرایط عدم قطعیت همراه با کدنویسی در نرم‌افزار (GAMS)

**Decision-Making under Uncertainty (Robust Optimization - Stochastic Programming - Fuzzy Programming)**

مدرس:

**دکتر علی پاپی (Ali Papi)**

تخصص شاخص: بهینه‌سازی و تحقیق در عملیات، علم تحلیل داده، تکنیک‌های تجزیه و روش‌های حل دقیق، بهینه‌سازی استوار داده‌محور، هوش محاسباتی و الگوریتم‌های فراابتکاری، نظریه بازی، بهینه‌سازی چندهدفه و تصمیم‌گیری چندمعیاره

Optimization & Operations Research, Data Analytics, Computational Intelligence & Metaheuristics, Decomposition Techniques & Exact Methods, Data-Driven Robust Optimization, Game Theory, Multi Criteria Decision Making